## VARIABILITY OF THE INFLATION RATE AND THE FORWARD PREMIUM IN A MONEY DEMAND FUNCTION: THE CASE OF THE GERMAN HYPERINFLATION

By: Stuart D. Allen and Donald L. McCrickard

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### Article:

### *I. INTRODUCTION*

Since Cagan's (1956) classic study, the specification of the demand for money in the German hyperinflation has attracted much attention. Cagan originally estimated an equation of the semi-logarithmic form:

$$m_t = \beta + \alpha\pi_t + u_t, \tag{1}$$

where $m_t$ is the log of real balances at time $t$, $\pi_t$ is the expected rate of inflation for period $t$ to $t + 1$ and $u_t$ is a random variable with a zero mean. A major difficulty of incorporating the role of expectations into empirical work is the lack of an observable variable measuring expectations. Cagan uses an adaptive expectations formulation which incorporates a distributed lag of current and past rates of inflation to solve for $\pi_t$:

$$\pi_t = (1-\psi)\sum_{i=0}^{\infty} \psi^i \log \frac{P_{t-1}}{P_{t-i-1}} \qquad 0 \leq \psi \leq 1, \tag{2}$$

where $P$ is the price level. Subsequent hyperinflation money demand studies by Sargent and Wallace (1973) and Sargent (1977) have restricted Cagan's adaptive expectations formula to yield rational expectations in the sense of Muth (1961) such that

$$\pi_t = E(X_{t+1} \mid \phi_t), \tag{3}$$

where $X_{t+1} = \log P_{t+1} - \log P_t$ and $E(X_{t+1})$ is the mathematical expectation of $X_{t+1}$ based upon the information set $(\emptyset_t)$ available in period $t$. These models allow for restrictions to be placed both upon $u_t$ of equation (1) and the stochastic process of money creation so that the model's prediction of expectations and Cagan's adaptive expectations are identical. Thus, the models take into consideration the feedback mechanism from inflation to money creation due to the government financing of expenditures.[1]

Frenkel (1976, 1977, 1979) implements an alternative approach to the problem of a non-observable measure of $\pi_t$. He uses the forward premium on foreign exchange as a rational expectations proxy variable for $\pi_t$.[2] If external assets are the major alternative to holding domestic money during hyperinflation, then the forward premium can serve as a rational expectations measure of the expected future depreciation of the currency and can be substituted for $\pi_t$ in Cagan's money demand equation. The validity of the argument depends on whether foreign exchange was a viable alternative to holding domestic money. Salemi (1980b) notes that Germany placed restrictions on the holding of foreign currency by its residents. If this restriction was effective, the use of the forward premium may have little power in a money demand function for Germany during the hyperinflation.[3]

Using the forward premium, $\lambda_t$, as a proxy for $\pi_t$, Frenkel (1979, table 1, eq. 5) estimates the money demand function for Germany from February 1921 to August 1923 and obtains the following results:

$$(4) \qquad m_t = \underset{(0.56)}{4.98} - \underset{(0.28)}{1.65}\ \lambda_t$$

$$R^2 = 0.97 \qquad D.W. = 1.66 \qquad S.E. = 0.11 \qquad \rho = 0.97$$

where standard errors are in parentheses. The significance of the forward premium coefficient provided some initial evidence to support Frenkel's argument that $\lambda_t$ could be employed as a rational expectations proxy variable.

Abel, Dornbusch, Huizinga and Marcus (hereafter ADHM, 1979) noted that if both foreign currency and domestic goods are alternatives to holding domestic money during a hyperinflation, then Frenkel's model is misspecified.[4] They suggest that both the expected rate of depreciation and the expected inflation rate should be included as explanatory variables. ADHM's apparent rationale for including both $\lambda_t$ and $\pi_t$ in their money demand equation stems from short run deviations from purchasing power parity. If such deviations exist, $\lambda_t$ measures the expected depreciation of the currency but not expected inflation *per se*. They estimate the following equation for February 1921 to August 1923:

$$(5) \qquad m_t = \underset{(0.50)}{7.45} - \underset{(0.08)}{0.59}\ \pi_t - \underset{(0.40)}{1.57}\ \lambda_t$$

$$R^2 = 0.98 \qquad D.W. = 1.73 \qquad S.E. = 0.10 \qquad \rho = 0.97$$

where standard errors are in parentheses and the forward premium, $\lambda_t$ is the logarithm of the forward rate in the current period minus the logarithm of the spot rate in the current period.

Although ADHM and Frenkel assume rational expectations, both use Fair's (1970) two-stage least squares method to estimate equations (4) and (5). This procedure is inconsistent with rational expectations because Fair's method includes only the lagged one-period dependent and independent variables in the money demand equation, time and time squared. Therefore, the information set is limited in time to variables in period *t* - 1 and in scope by the omission of other relevant variables. Recent empirical work by Salemi and Sargent (1979) and Salemi (1980b) which assumes rational but not necessarily adaptive expectations, employs an information set which includes the rate of money creation, exchange depreciation and inflation.

We assume that equilibrium values and policy decisions in period *t* are not available until the end of the period so that the public forms expectations in period *t* on the basis of $\emptyset_{t-1}$. The information set, $\emptyset_{t-1}$, which is implemented by instrumental variables in this study includes past rates of inflation, monetary growth, currency depreciation as measured by the forward premium, time and time squared. Therefore equation (3) is rewritten as equation (6):

$$(6) \qquad \pi_t = E(X_{t+1} \mid \phi_{t-1}).$$

ADHM also employ the logarithm of the rate of inflation, $X_t = \log P_t - \log P_{t-1}$, for $\pi_t$ rather than employing $X_{t+1}$, the future expected rate of inflation, for $\pi_t$ as in equation (3).[5] This definition of $\pi_t$ is inconsistent with their rational expectations assumption. We substitute both $X_t$ and $X_{t+1}$ for $\pi_t$ with the reported results based upon using $X_t$ for $\pi_t$ in the first step of the instrumental variables procedure.[6] Therefore, our model is motivated by, but is not equivalent to rational expectations models.

In section II we criticize Frenkel's and ADHM's estimates of equations (4) and (5). Evidence is presented that shows the coefficient of $\lambda_t$ is always insignificant in equation (4) except when the sample period includes the August 1923 observation and is significant but its coefficient exhibits instability in equation (5). The first difference form of equations (4) and (5) are tested in section III to provide additional verification of these conclusions. In section IV money demand equations which include measures of *both* the level and variability of the inflation rate and the expected foreign exchange depreciation are tested. Final remarks about our results and suggestions for future research are noted in section V.

**TABLE 1**

Demand for Money Estimates with an Instrumental Variables Approach

| | EQUATION (4) | | | | | | EQUATION (5) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Period** | **Eq.** | $\beta$ | $\lambda$ | **S.E.** | **DW** | $\rho$ | **Eq.** | $\beta$ | $\lambda$ | $\pi$ | **S.E.** | **DW** | $\rho$ |
| 12/21 – 2/23 | (1.1) | 7.55<br>(9.18) | −2.85<br>(1.26) | .1842 | 1.16 | .98 | (1.8) | 8.26<br>(21.98) | −4.18<br>(4.00) | −1.01<br>(6.41) | .0831 | 1.39 | .97 |
| 12/21 – 3/23 | (1.2) | 8.08<br>(10.24) | 0.65<br>(0.44) | .2086 | 1.36 | .96 | (1.9)* | 8.12<br>(11.64) | −4.19<br>(6.46) | −1.19<br>(7.95) | .0844 | 1.73 | .99 |
| 12/21 – 4/23 | (1.3) | 8.14<br>(16.04) | −0.65<br>(0.59) | .2209 | 1.57 | .92 | (1.10) | 7.65<br>(8.08) | −2.15<br>(4.66) | −0.91<br>(7.32) | .0845 | 0.64 | .99 |
| 12/21 – 5/23 | (1.4) | 8.13<br>(16.27) | −0.69<br>(0.64) | .2144 | 1.64 | .93 | (1.11) | 8.09<br>(15.62) | −1.99<br>(3.90) | −0.83<br>(6.37) | .0945 | 0.85 | .98 |
| 12/21 – 6/23 | (1.5) | 7.94<br>(15.77) | −0.94<br>(0.88) | .2157 | 1.70 | .93 | (1.12) | 8.17<br>(17.43) | −1.57<br>(2.87) | −0.64<br>(4.54) | .1082 | 0.72 | .97 |
| 12/21 – 7/23 | (1.6)* | 7.76<br>(7.94) | −0.67<br>(0.48) | .2790 | 1.18 | .96 | (1.13) | 7.93<br>(12.30) | −1.69<br>(3.25) | −0.77<br>(8.57) | .1056 | 0.95 | .98 |
| 12/21 – 8/23 | (1.7)* | −8.89<br>(3.15) | −2.55<br>(2.42) | .3067 | 1.70 | .99 | (1.14) | 8.05<br>(13.87) | −1.10<br>(2.64) | −0.70<br>(9.00) | .1115 | 1.08 | .97 |

T-scores are in parentheses. The regressions were run on the 3.5c version of TSP. The * equations utilized the Hildreth-Lu procedure to search for the $\rho$ value which would minimize the standard error. Because the procedure drops the first observations, the results are reported for the MLGRID procedure which retains the first observation with a special weight.

## II. RE-EXAMINATION OF FRENKEL'S AND ADHM'S RESULTS

The purpose of this section is to evaluate the results of Frenkel and ADHM by reestimating equations (4) and (5) .[7] We employ the Brown-Durbin-Evans technique (1975) to evaluate equations (4) and (5) for their sensitivity to the sample period. The equations are initially estimated for the December 1921-February 1923 time period with a successive month added to each subsequent regression. The results are presented in table 1. The forward premium coefficient in equation (4) is not significant except for the December 1921-August 1923 sample period for which ADHM found λ to be significant. These results clearly are very sensitive to the sample period examined, particularly to the inclusion of the August 1923 data point. Frenkers use of the forward premium as the only rational expectations measure is obviously without firm foundation.

The estimates of equation (5) reported in table 1 confirm ADHM's results: the coefficients on λ and π it are negative and significant and the standard error of equation (5) is approximately one-half that of equation (4). However, the forward premium coefficient in equation (5) displays instability, rising from -4.19 to -1.10 as the sample period is extended.[8] The instability of the forward premium is but one aspect of equation (5) which suggests that the time series is not stationary. The Durbin-Watson statistics are low even after the correction for autocorrelation by a rho which is not significantly different than one.

The instability of λ was also verified by estimating equation (7) which includes variables to capture coefficient drift over time. These variables include a time trend variable *(T)* numbered one to twenty-one for the December 1921-August 1923 period and time-trend-interaction terms formed by multiplying *T* by the expected inflation rate ($\pi T$) and the forward premium ($\lambda T$). We omitted $\pi T$ because it was insignificant.

$$
(7) \qquad m = \underset{(106.5)}{9.23} - \underset{(9.56)}{.048}\, T - \underset{(0.12)}{21.29}\, \lambda_t + \underset{(8.15)}{.703}\, \lambda T - \underset{(11.02)}{.764}\, \pi_t .
$$

S.E. = .0966 D.W. = 1.65 $\rho = -.260$

T-scores are in parentheses. The significance of the time trend, $T$, confirms a continuous shift in the intercept term while the significance of $\lambda T$ confirms coefficient drift for $\lambda$.[9] The results lead inevitably to the conclusion that there is not just a structural shift, but that there is a continuous drift in the constant term and the $\lambda$ coefficient during the hyperinflation. The instability of the money demand function suggests either that other variables may have been omitted or that the structure yielding the data is not stationary.[10] Therefore, in section III we will report the first-difference forms of equations (4) and (5) which verify our conclusions from this section. The question of omitted variables is deferred until section IV where we introduce variability terms into the money-demand equation.

## III. FIRST-DIFFERENCE RESULTS

Recent work by Plosser and Schwert (1977, 1978) has shown that taking the first difference of an equation eliminates a linear time trend from each of the right-hand-side variables and results in unbiased coefficient estimates. Given the evidence of nonstationarity cited in section II, first-difference forms of equations (4) and (5) (labeled (4') and (5')) were estimated:

$$\Delta m_t = \Delta\lambda_t + \epsilon_{1t} \qquad (4')$$

$$\Delta m_t = \Delta\lambda_t + \Delta\pi_t + \epsilon_{2t}, \qquad (5')$$

where $\Delta m_t = m_t - m_{t-1}$; $\Delta\lambda_t$ and $\Delta\pi_t$ are analogously defined; and $\varepsilon_{it} = u_{it} - \xi u_{i,t-1}$ with $i = 1, 2$, and $\xi = 1$.

**TABLE 2**

First-Difference Estimates

| | EQUATION (4')* | | | | | EQUATION (5')** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Period | Eq. | $\Delta\lambda$ | S.E. | DW | $\rho$ | Eq. | $\Delta\lambda$ | $\Delta\pi$ | S.E. | DW | $\rho$ |
| 12/21 – 2/23 | (2.1) | 0.57<br>(0.36) | .148 | 1.55<br>(0.88) | 0.61<br>(2.61) | (2.8) | –2.05<br>(3.95) | –0.79<br>(9.40) | .046 | 1.52<br>(1.51) | 0.82<br>(6.34) |
| 12/21 – 3/23 | (2.2) | 3.17<br>(2.10) | .177 | 2.10<br>(1.23) | 0.45<br>(2.06) | (2.9) | –2.07<br>(3.69) | –0.86<br>(11.92) | .048 | 1.66<br>(1.59) | 0.77<br>(5.26) |
| 12/21 – 4/23 | (2.3) | 0.72<br>(0.90) | .198 | 1.84<br>(1.38) | 0.38<br>(1.51) | (2.10) | –1.69<br>(7.14) | –0.83<br>(15.99) | .046 | 1.74<br>(0.48) | 0.80<br>(6.13) |
| 12/21 – 5/23 | (2.4) | 0.26<br>(0.31) | .196 | 1.94<br>(1.59) | 0.25<br>(1.09) | (2.11) | –1.24<br>(5.79) | –0.72<br>(14.15) | .056 | 1.44<br>(0.69) | 0.82<br>(6.06) |
| 12/21 – 6/23 | (2.5) | –0.59<br>(0.64) | .202 | 1.93<br>(1.81) | 0.14<br>(0.58) | (2.12) | –1.15<br>(5.00) | –0.66<br>(10.92) | .061 | 1.76<br>(0.79) | 0.80<br>(5.92) |
| 12/21 – 7/23 | (2.6) | –0.14<br>(0.14) | .254 | 1.51<br>(1.38) | 0.39<br>(1.39) | (2.13) | –1.31<br>(4.11) | –0.76<br>(8.77) | .077 | 1.33<br>(0.86) | 0.63<br>(3.55) |
| 12/21 – 8/23 | (2.7) | –0.65<br>(0.85) | .268 | 1.86<br>(1.88) | 0.57<br>(2.67) | (2.14) | –0.81<br>(3.01) | –0.66<br>(7.94) | .092 | 1.93<br>(1.08) | 0.51<br>(2.76) |
| 12/21 – 8/23 | (2.7)' | –2.84<br>(2.72) | .300 | 1.88 | .... | | | | | | |

The regressions were run with the 3.5c version of TSP. T-scores are in parentheses except for the term below the DW statistic which is the value of DW for the equation if there is no correction for autocorrelation by the Beach-MacKinnon procedure.

* The coefficient for $\Delta\lambda$ is insignificant in every case when the equation is not corrected for autocorrelation.

**The coefficient for $\Delta\lambda$ and $\Delta\pi$ are negative and insignificant when the equation is not corrected for autocorrelation.

Equations (4') and (5') were estimated with the results reported in table 2. The evidence confirms the conclusions of section II. The forward premium is insignificant when it is used as the sole variable to capture inflationary expectations. The only exception is the December 1921-August 1923 period [see equation (2.7')] where $\lambda$ is negative and significant when there is no correction for autocorrelation. The forward premium is insignificant in every other estimate of equation (4') whether the estimates are corrected for autocorrelation by the Beach-MacKinnon (1978) technique or not. The significance of rho, the autocorrelation correction factor in both equations (4') and (5'), is evidence of second-order autocorrelation in the level equations (equations (4) and (5) ).[11]

The forward premium coefficient is negative and significant in equation (5'), though it again exhibits coefficient drift since $\lambda$ rises from -2.07 to -0.81. The $\pi$ coefficient is also negative and significant and is more stable than in the level equation. The standard errors are reduced by more than 70 percent between equations (4') and (5') when both $\pi_t$ and $\lambda_t$ are included in a reduced-form money demand function. These results are very similar to those of the level equation which increases our confidence in the main conclusions: (1) that the forward premium cannot be employed as the only measure of the expected rate of inflation in the empirical work on the German hyperinflation, and (2) that the forward premium coefficient is not stable in an equation which uses both the forward premium and the expected rate of inflation as right-hand-side variables.

## IV. THE VARIABILITY OF INFLATIONARY EXPECTATIONS AND OF THE FORWARD PREMIUM

In this section we test for the significance of two additional variables which have been omitted from the money demand specifications during hyperinflation. Several money demand studies have assessed the impact of the inflation rate variability on the holdings of real money balances. Khan (1977) hypothesizes that the coefficient of $\pi_t$ may vary over time and be a linear function of both the absolute value and the variability of the rate of inflation. He finds that (p. 824) "the variability of inflation has a more consistent [positive] influence on the change in expectations than does the level [of inflation]."

Blejer (1979) has proposed that an inflation variability measure be included in the demand-for-money specification as a proxy for uncertainty about the future rate of inflation. Blejer notes that the effect of increased uncertainty on money demand is theoretically ambiguous. This point had been made by Matthews (1963) and Frenkel (1977).[12] As Frenkel (p. 661) notes, ". . . a higher variance [of inflation] may raise the degree of uncertainty and thereby raise the precautionary demand [for money]. On the other hand, the variability of price may reduce the usefulness of money as a unit of exchange and thereby reduce the extent to which the economy is monetized." This issue is further complicated because (Frenkel, p. 662) "... the concept of variability need not coincide with that of uncertainty."[13]

While the variability of past inflation rates may be a poor guide to future inflation variability during a hyperinflation, two measures of the variability of the domestic rate of inflation ($V_\pi$) and of the rate of depreciation of foreign exchange ($V_\lambda$) were tested. A measure of the variability of $\pi_t(BV_\pi)$ which is similar to Blejer's measure is defined as the average absolute value of the change in the rate of inflation over *n* lagged periods:

$$BV_\pi = \frac{1}{n}\sum_{i=1}^{n} | X_{t-i} - X_{t-i-1} | \tag{8}$$

where $n = 6$. A measure for the variability of the forward premium $(BV\pi)$ is similarly defined. An alternative measure of the variability of $\pi_t(KV_\pi)$, which is similar to Klein's (1977) price uncertainty term, is defined as the *n*-term moving standard deviation from an *n*-term moving average of the monthly inflation rate:

$$KV_\pi = \frac{1}{n}\sum_{i=1}^{n} ((X_{t-i} - \overline{X})^2)^{1/2} \tag{9}$$

where $n = 6$. A measure for the variability of the forward premium $(KV_\pi)$ is similarly defined. Therefore, the first-difference form of the demand-for-money function becomes:

$$\Delta m_t = \theta\Delta\lambda_t + \delta\Delta V_\lambda + \alpha\Delta\pi_t + \psi\Delta V_\pi + \epsilon_t, \tag{10}$$

where $\epsilon_t = u_t - u_{t-1}$, and the expected coefficient signs are $\theta, \alpha < 0$ and $\delta, \psi \gtrless 0$. The sign and significance of the variability terms are theoretically indeterminant and may depend upon the severity of the inflation and the public's response to it.[14] If greater price variability increases the financial risk of holding money as a medium of exchange, then $V_\pi$, the risk associated with holding domestic money balances, would be negative. If foreign exchange is also a relevant substitute for holding domestic money at the external (rather than internal) margin of substitution, it should also have a negative sign. If $V_\lambda$ is the risk of holding an alternative asset such as foreign exchange, then $V_\lambda$ would be positive.[15]

**TABLE 3**

First-Difference Estimates with Blejer's Variability Measures

| Period | Eq. | $\Delta\lambda$ | $\Delta BV_\lambda$ | $\Delta\pi$ | $\Delta BV_\pi$ | SE | DW | $\rho$ | $\theta = \psi = 0$ |
|---|---|---|---|---|---|---|---|---|---|
| 12/21 – 2/23 | (3.1) | −2.74 (4.27) | 3.14 (0.38) | −0.97 (7.55) | −1.58 (1.68) | .046 | 1.61 (1.04) | 0.75 (4.40) | 1.16 (3.98) |
| 12/21 – 3/23 | (3.2) | −2.81 (4.27) | 2.46 (0.30) | −1.05 (7.51) | −1.51 (1.44) | .050 | 1.65 (1.13) | 0.66 (3.27) | 0.50 (3.89) |
| 12/21 – 4/23 | (3.3) | −3.13 (5.52) | 1.68 (0.30) | −1.18 (8.15) | −2.78 (2.23) | .052 | 1.80 (1.36) | 0.46 (1.98) | 0.79 (3.81) |
| 12/21 – 5/23 | (3.4) | −3.23 (5.75) | 13.04 (3.31) | −1.35 (8.65) | −5.00 (3.82) | .066 | 1.89 (1.60) | 0.20 (0.76) | 1.27 (3.74) |
| 12/21 – 6/23 | (3.5) | −0.85 (1.17) | 0.17 (0.05) | −0.62 (4.47) | 0.90 (0.70) | .062 | 1.64 (0.47) | 0.82 (6.45) | 0.32 (3.68) |
| 12/21 – 7/23 | (3.6) | −2.30 (3.82) | −2.54 (0.61) | −0.94 (8.97) | −1.95 (1.99) | .075 | 1.49 (0.82) | 0.77 (5.50) | 1.65 (3.63) |
| 12/21 – 8/23 | (3.7) | −0.69 (1.99) | 2.48 (0.54) | −0.71 (9.36) | 0.60 (1.16) | .089 | 2.04 (0.92) | 0.61 (3.38) | 1.79 (3.59) |

The regressions were run with the 3.5c version of TSP. T-scores are in parentheses except for the term below the DW statistic which is the value of DW for the equation if there is no correction for autocorrelation by the Beach-MacKinnon procedure.

**TABLE 4**

First-Difference Estimates with Klein's Variability Measures

| Period | Eq. | $\Delta\lambda$ | $\Delta KV_\lambda$ | $\Delta\pi$ | $\Delta KV_\pi$ | S.E. | DW | $\rho$ |
|---|---|---|---|---|---|---|---|---|
| 12/21 – 2/23 | (4.1) | −2.52 (4.01) | 5.62 (2.90) | −0.90 (8.90) | −0.73 (4.27) | .0501 | 1.49 | .... |
| 12/21 – 3/23 | (4.2) | −2.64 (4.61) | 5.25 (3.20) | −0.89 (12.55) | −0.69 (3.69) | .0467 | 1.69 (1.47) | 0.26 (0.96) |
| 12/31 – 4/23 | (4.3) | −1.73 (6.28) | 2.02 (1.17) | −0.84 (14.41) | −0.42 (1.52) | .0491 | 1.69 (1.12) | 0.59 (2.83) |
| 12/31 – 5/23 | (4.4) | −1.59 (6.79) | 2.89 (3.46) | −0.79 (14.53) | −0.53 (2.79) | .0983 | 1.73 (1.06) | 0.53 (2.52) |
| 12/31 – 6/23 | (4.5) | −1.61 (6.98) | 2.67 (3.73) | −0.79 (14.80) | −0.48 (2.83) | .0474 | 1.78 (1.10) | 0.55 (2.73) |
| 12/31 – 7/23 | (4.6) | −2.00 (5.36) | 3.06 (4.25) | −1.03 (12.02) | −0.61 (3.82) | .0733 | 1.90 | .... |
| 12/31 – 8/23 | (4.7) | −1.13 (3.31) | 2.38 (2.78) | −0.84 (10.21) | −0.47 (2.48) | .0907 | 1.94 | .... |

The regressions were run with the 3.5c version of TSP. T-scores are in parentheses except for the term below the DW statistic which is the value of DW for the equation if there is no correction for autocorrelation by the Beach-MacKinnon procedure.

The results of estimating equation (10) with the Blejer variability terms are reported in table 3, while equation (10) with the Klein variability terms are reported in table 4. These results confirm the expected negative and significant coefficients for $\lambda$ and $\pi$. While the $\lambda$ coefficient is significant, it still exhibits coefficient drift from -2.64 to -1.13 in the Blejer version of equation (10) and from -3.28 to -1.04 in the Klein version. The instability of the forward premium in equation (10) suggests the existence of coefficient drift despite the inclusion of the variability terms.

The coefficients of the Blejer variability measures for the forward premium ($BV_\lambda$) are insignificant in each case while the coefficient for the variability of the inflation rate ($BV_\pi$) is significant at the five percent level for a two-tailed test for only two of the tested sample periods. A joint test that $\delta$ and $\psi$ are equal to zero was made but we could not reject the null hypothesis. The calculated F-statistics are presented in the last column of table 3 with the appropriate critical value in parenthesis under the test statistic. The coefficients of both Klein variability measures, however, are significant at the five percent level for a two-tailed test in six of the seven periods. These inconsistent results concerning the significance of the Blejer and Klein variability measures are surprising because the correlation between $KV_\pi$ and $BV_\pi$ is 0.921 and the correlation between $KV_\lambda$ and $BV_\pi$ is 0.925. These results further point out the sensitivity of the results for the German hyperinflation. What would appear to be minor differences in these two variability measures result in different conclusions concerning their significance.

## V. CONCLUSIONS

The purpose of the paper has been to reexamine the empirical findings of Frenkel and ADHM on the money demand equation during the German hyperinflation. We have provided evidence of the sensitivity of the results reported by Frenkel and ADHM with respect to the sample period and the equation specification. Specifically, the evidence shows that the forward premium ($\lambda_t$) is insignificant in all but one of the sample periods when it is the only measure of the expected rate of inflation. When both $\lambda_t$ and the inflation rate ($\pi_t$) are included in the specification, each is negative and significant. The $\lambda$ coefficient, however, tends to drift upward as the sample period is extended. It also displays similar behavior when the variability of the forward premium ($V_\lambda$) and the variability of the inflation rate ($V_\pi$) are included in the money demand equation.

The equations which include the variability terms, $V\lambda$ and $V_\pi$ , provide further evidence of the sensitivity of the results. The evidence shows that variability terms for $KV_\lambda$ and $KV_\pi$ , defined in a manner similar to Klein's technique, are significant while the variability terms $BV_\lambda$ and $BV_\pi$ , defined in a manner similar to Blejer's technique, are insignificant despite the high correlations between the variability measures. The evidence of significant coefficient estimates for the variability terms is an important finding but our demonstration of the sensitivity of the econometric findings suggests caution in drawing conclusions. Further research will have to establish the robustness of these results for the German hyperinflation and for other hyperinflations.

## Notes:

1. Whereas Cagan's original model is subject to simultaneous equation bias and inconsistent estimates of $\alpha$, these rational expectations models can estimate $\alpha$ when an added assumption is imposed about the [Sargent (1977, p. 611)] "covariance of the disturbances to the demand for money and to the supply of money."
2. The use of the forward premium on foreign exchange as a rational expectation measure of $\pi_t$ is debated by Salemi (1980a) and Frenkel (1980).
3. During the German hyperinflation the forward exchange rate not only underpredicted the future spot rate (Einzig, p. 289) but also was selling at a forward premium prior to July 1922. Previously, Frenkel (1977, 1979) and Taylor (1975) have noted that this fact need not be inconsistent with rational expectations during a transition to hyperinflation.
4. This approach was originally suggested by Holtfrerich (1976).
5. See ADHM, pp. 102-103 for a discussion of their data and definition of $\pi_t$.
6. A referee suggested that $X_{t+1}$ be employed as the basis for estimating $\pi_t$ rather than $X_t$. This procedure, however, usually produced insignificant coefficients for $\pi_t$.

7. The instrumental variables were also extended to three lagged periods with no significant effect upon the results.
8. There is additional evidence of the instability of the forward premium. Frenkel (1977) considered the efficiency of the foreign exchange market by examining the following equation:

$$\log S_t = a + b \log F_{t-1} + u_t ,$$

where $S_t$ is the current spot exchange rate and $F_{t-1}$ is the one month forward premium in the previous month. If the foreign exchange market is efficient, all available information will be reflected in the current forward premium. One test of efficiency would hypothesize that a is equal to zero and *b* is equal to one while the error term ($u_t$) is serially uncorrelated. Frenkel employed a Chow test with subperiods of moderate hyperinflation February 1921-November 1922 and severe hyperinflation December 1922-August 1923 and failed to reject the null hypothesis of structural stability at the five percent level of significance. We confirm Frenkel's result for this division using ADHM's data. However, the F-statistic for a Chow test where the subperiods were divided between June and July 1922, the month the mark shifted from a premium to a discount in terms of sterling, is equal to 29.19. At the one percent significance level the critical value of $F_{2,27}$ is 5.49. Therefore, the null hypothesis of structural stability of the above equation is easily rejected.
9. The correlation coefficient between $\lambda$ and $\lambda T$ and $\pi$ and $\pi T$ is .890 and .950 respectively. The existence of multicollinearity, however, does not bias the coefficient estimates.
10. Cagan recognized that real income should be included in the money demand function, but data limitations require that it be assumed constant. Since during hyperinflation changes in nominal quantities dwarf changes in real quantities, this should not be a significant omission.
11. There is no evidence of higher than second-order autocorrelation because an equation which regressed the error terms against lagged error terms for up to a four period distributed lag revealed no significant coefficients.
12. We are indebted to W. James Smith for the Matthews reference.
13. Frenkel uses the prediction error as the best available measure of the uncertainty of holding money balances during a hyperinflation. He examines the variance of the prediction error between the ex-ante forward premium ($\ln F_{t-1} - \log S_{t-1}$) and the ex-post change ($\ln S_t - \ln S_{t-1}$) and finds an insignificant coefficient for this measure of uncertainty.
14. Blejer finds a negative and significant coefficient for a variant of V, for the rapid inflations of Argentina, Brazil and Chile. Klein (1977), using his measure of variability, finds evidence of a positive and significant coefficient for the price uncertainty term for the United States. His results have been challenged by Laidler (1980, pp. 230-2) and Allen (forthcoming). Some evidence of negative and significant coefficients for the Klein measure have been found for the German hyperinflation by Allen (1979) and Pautler (1981).
15. A referee has argued that the sign of $V_x$ should be negative because foreign currency was clearly acceptable if not preferred in domestic transactions by the end of the hyperinflation.

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